Time-dependent tube flow of compressible suspensions subject to pressure dependent wall slip: Ramifications on development of flow instabilities

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(Received 1 August 2007; final revision received 19 May 2008)

Synopsis

A mathematical model developed earlier for the time-dependent circular tube flow of compressible polymer melts subject to pressure-dependent wall slip [Tang and Kalyon, J. Rheol \textbf{52}, 507–525 (2008)] was applied to the tube flow of polymeric suspensions with rigid particles. The model relies on the apparent slip mechanism for suspension flow with the additional caveat that the polymeric binder slips at the wall according to a pressure-dependent wall slip condition. The numerical simulations of the tube flow of concentrated suspensions suggest that steady flow is generated when the flow boundary condition at the wall is a contiguous strong slip condition along the entire length of the tube wall. The findings of the simulations are consistent with the experimental flow curves and flow instability data collected on suspensions of a poly(dimethyl siloxane), which itself exhibits wall slip, compounded with rigid and hollow spherical particles in the 10–40\% by volume range. Increasing the concentration of rigid particles gives rise to the expansion of the range of flow rates over which the flow remains stable, as consistent with the experimental observations. © 2008 The Society of Rheology. [DOI: 10.1122/1.2955508]

I. INTRODUCTION

The development of flow instabilities during the flow of concentrated suspensions of rigid particles incorporated into polymeric binders in simple capillary dies (cylindrical reservoir connected to a converging entry region into a straight circular tube) can involve time-dependent changes in the concentration of particles or changes in the shape of the extrudates emerging from tube flow. Time-dependent changes in the concentration of particles generally occur on the basis of time-periodic oscillations in extrusion pressure, upon the formation and break-up of mats of solids at the capillary entrance and the filtration of the binder phase [Yaras et al. (1994); Rough et al. (2000)]. The time-dependent changes in the concentration of rigid particles of extrudates of concentrated suspensions become especially significant with decreasing binder shear viscosity (filtration based flow instabilities are especially prevalent with Newtonian binders), increasing particle size, increasing capillary convergence ratio (reservoir diameter over the tube diameter), and decreasing flow rate [Yilmazer et al. (1989)]. On the other hand, with increasing shear viscosity of the binder phase, concentrated suspensions exhibit time and apparent shear rate-dependent changes in the shape of extrudates emerging from tube

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flow [Birinci and Kalyon (2006)] akin to flow instabilities associated with various types of extrudate shape distortions observed in the tube flow of polymeric melts [Benbow and Lamb (1963); Kalika and Denn (1987); Denn (2001); Robert et al. (2004); Hatzikiriakos and Migler (2005)].

The effects of the presence and the concentration of rigid particles on the development of shape distortions of the extrudates of suspensions are pronounced and contradictory. For an elastomer that does not exhibit flow instabilities over a wide range of shear rates in capillary or rectangular slit flows, the incorporation of particles induced extrudate shape distortions over the same apparent shear rate range [Tang and Kalyon (2004a)]. On the other hand, for other polymer melts that exhibit instabilities upon flow through cylindrical dies, the incorporation of particles reduces the range of apparent shear rates over which flow instabilities are observed at relatively low particle concentrations [Rosenbaum et al. (2000); Birinci and Kalyon (2006)] and eliminates them altogether at relatively high particle concentrations [Birinci and Kalyon (2006)]. Currently, there is no explanation for this type of contradictory behavior involving the induction or elimination of flow instabilities upon the incorporation of particles into polymeric melts.

The slip-based theoretical treatment of the development of flow instabilities of polymer melts in simple channels has generally relied on the considerations of the compressibility and various empirical nonmonotonic wall slip velocity versus wall shear stress expressions [Hatzikiriakos and Dealy (1992a); Den Doelder et al. (1998); Georgiou (2003)]. For polymeric liquids the pressure dependence of the wall slip velocity can be significant [Vinogradov and Ivanova (1967); Hatzikiriakos and Dealy (1992b); Person and Denn (1997); Tang and Kalyon (2008)] with the slip velocity decreasing with increasing pressure. Since the flow curves characterized over relatively high apparent shear rates are subject to wall slip, and since such slip is a function of wall shear stress, as well as pressure, new methodologies, including the use of inverse problem solutions and squeeze flow, are necessary for the characterization of the parameters of pressure-dependent wall slip and shear viscosity material functions [Tang and Kalyon (2008)].

With suspensions the primary cause of wall slip is the apparent slip mechanism associated with the formation of a pure binder or binder-rich apparent slip layer at the wall [Reiner (1960)]. In the following, first the analysis of the apparent slip mechanism of concentrated suspensions is expanded to include the hitherto-neglected slip of the polymeric binder at the wall. Second, a mathematical model and corresponding numerical simulation results of the time-dependent isothermal tube flow of compressible suspensions with rigid spherical particles, subject to pressure-dependent wall slip, are provided and applied to a model suspension at differing concentrations of rigid particles. Third, procedures for the experimental determination of the parameters of pressure-dependent wall slip and shear viscosity material function for suspensions (based principally on the inverse-problem solution methodologies) are presented. Finally, the capillary flow data of suspensions of a poly(dimethyl siloxane), PDMS, incorporated with rigid spherical glass particles, collected with relatively long capillary dies, are compared directly with the time-dependent numerical simulation results of tube flow to elucidate the underlying mechanisms for the development of flow instabilities for concentrated suspensions.

II. APPARENT WALL SLIP BEHAVIOR OF CONCENTRATED SUSPENSIONS WITH RIGID PARTICLES

The wall slip behavior of the concentrated suspensions of rigid particles with relatively low aspect ratios, incorporated into a non-Newtonian binder, is considered to occur on the basis of the apparent slip mechanism subject to the wall slip of the polymeric
binder (Fig. 1). The apparent slip layer (Vand layer, i.e., a zone which is free of particles and thus consists solely of the binder) has a thickness, $\delta$, at the wall. Since the thickness of the slip layer, $\delta$, would be significantly smaller than the channel gap, the formation of the slip layer gives the appearance of wall slip; hence the “apparent slip” at the wall [Reiner (1960); Cohen and Metzner (1985); Yilmazer and Kalyon (1989); Kok Hartman et al. (2002); Tabuteau et al. (2004); Kalyon (2005)]. This mechanism is depicted in an exaggerated manner in Fig. 1, which shows the apparent slip layer formed next to the wall of the tubular die. Such apparent wall slip may or may not be significant in comparison to the mean velocity in the channel, i.e., constituting the weak versus strong wall slip of the suspension. The rigidity of the particles can alter the dynamics of the formation of the apparent slip layer [Meeker et al. (2004); Adams et al. (2004)].

Unlike the earlier analyses in the treatment of apparent slip, here the binder itself is also considered to be subject to a slip condition with the wall slip velocity of the polymeric binder, $u_{sb}$, following a hyperbolic tangent-type dependence on the wall shear stress, $\tau_w$ [Tang and Kalyon (2004a); Tang and Kalyon (2008)]:

$$u_{sb} = b \tau_w^{s_1}(0.5 + 0.5 \tanh(\alpha(\tau_w - \tau_c)))$$

where $b$ and $s_1$ are the slip coefficient (referred to as Navier’s slip coefficient for $s_1 = 1$) and slip exponent of the polymeric binder, respectively, and $\alpha$ is a positive constant (typically 1–20) describing the sharpness of the weak-to-strong slip transition in the slip velocity of the polymeric binder at the critical wall shear stress, $\tau_c$.

As noted by Hatzikiriakos and Dealy (1992b), for viscoelastic polymer melts the slip velocity should be a function of the pressure and the first and second normal stress differences at the wall. For polymeric suspensions with rigid particles the development of normal stress differences would also be affected by the formation of anisotropic particle clusters [Nott and Brady (1994); Zarraga et al. (2000)]. For the materials considered in our experimental study the normal stress effects were determined to be small and only the pressure effect is considered in our analysis. The slip coefficient for the binder, $b$, is assumed to vary inversely with pressure, akin to the slip of compressible gases during flow through simple conduits [Knudsen (1950)].
\[ \beta_b = \beta_0 \left( \frac{p_a}{p} \right)^\kappa, \]  

where \( p \) is pressure at any location in the die, \( p_a \) is the atmospheric pressure, \( \beta_0 \) is the slip coefficient of the binder at atmospheric pressure. The exponent \( \kappa \) becomes equal to one for Knudsen flow but needs to be determined experimentally for a polymeric melt [Tang and Kalyon (2008)] and its suspensions with rigid particles with \( \kappa \) dependent on the concentration of particles, i.e., \( \kappa(\phi) \). The available experimental data for polymer melts [Vinogradov and Ivanova (1967); Hatzikiriakos and Dealy (1992b)] indicate that the slip coefficient decreases with increasing pressure (thus \( \kappa \) is positive). The principal underlying mechanisms for pressure dependence of wall slip are considered to be the entrainment of air into the binder phase of the suspension and the residence time dependence of the establishment of the apparent slip condition as discussed later.

It is assumed that the Ostwald-de Waele or “power law” behavior represents the behavior of the shear viscosity of the binder phase in tube flow, i.e., \( \tau_{zc} = -m_b (du/dr)_{s_b-1} (du/dr) \), where \( u(r) \) is the axial \( z \) velocity and \( m_b \) and \( n_b \) are the consistency index and the power law index parameters of the Ostwald-de-Waele “power-law” equation for the pure binder, respectively. The slip velocity, \( u_s \), for the apparent slip mechanism [Kalyon, (2005)] subject to the slip of the binder at the tube wall (Fig. 1) consists of the contributions of the slip of the binder, \( u_{sb} \), and the contribution of the apparent slip mechanism, \( u_{sa} \):

\[ u_s = u_{sa} + u_{sb} = \frac{\tau_{zw}^b}{m_b^b (s_b + 1)} \left[ 1 - \left( 1 - \frac{\delta}{R} \right)^{s_b + 1} \right] + u_{sb}. \]  

Considering that the reciprocal power-law index of the binder, i.e., \( s_b = 1/n_b \), is positive and assuming an integer value—the use of Binomial Theorem provides \((1 - \delta/R)^{s_b+1} \equiv 1 - (s_b + 1) \delta/R\), the slip velocity at the interface between the apparent slip layer and the bulk of the suspension, \( u_s \), i.e., at \( r = R - \delta \), becomes:

\[ u_s = u_{sa} + u_{sb} = \frac{\tau_{zw}^b}{m_b^b} \delta + u_{sb} = \beta_s \tau_{zw}^b + u_{sb}, \]  

where \( \beta_s \) is the Navier’s slip coefficient resulting from the apparent slip mechanism, i.e., \( \beta_s = \delta/m_b^b \) [Kalyon (2005)]. A correlation for the apparent slip layer thickness, \( \delta \), and the Navier’s slip coefficient \( \beta_s = (D_p/m_b^b)(1 - \phi/\phi_m) \) can be used [Kalyon (2005)], where \( D_p \) is the particle diameter, \( \phi \) is the volume percent of rigid particles, and \( \phi_m \) is the maximum packing fraction of solids. This correlation for the mean value of the slip layer thickness (over the length of the die) applies for concentrated suspensions with rigid particles exhibiting low aspect ratios at \( \phi < \phi_m \) and suggests that the flow of the concentrated suspension approaches plug flow as \( \phi \rightarrow \phi_m \). This is seen below from the simplified analysis of the apparent wall slip of a Newtonian suspension with viscosity, \( \mu_s \) and with a Newtonian binder with viscosity \( \mu_b \) and on the basis of a no-slip condition at the wall for the Newtonian binder, which constitutes the apparent slip layer. The total flow rate, \( Q \), versus the wall shear stress, \( \tau_w \), relationship for the apparent slip flow of a Newtonian suspension in a tube with radius, \( R \), becomes:
\[ Q = Q_s + \frac{4\mu_s}{\pi R^3 \tau_w} = \pi R^2 u_s + \frac{\pi R^3 \tau_w}{4\mu_s} = \pi R^2 \frac{\delta}{\mu_b} \tau_w + \frac{\pi R^3 \tau_w}{4\mu_s}, \quad (5) \]

where \( Q_s = \pi R^2 (\delta/\mu_b) \tau_w \). The ratio of the flow rate due to slip \( Q_s \) over the total flow rate, \( Q \), i.e., \( Q_s/Q \), indicates the relative importance of wall slip at any wall shear stress, \( \tau_w \), and is given by:

\[ \frac{Q_s}{Q} = \frac{\delta}{\mu_b} \left( \frac{\delta}{\mu_b} + \frac{R}{4\mu_s} \right) = \frac{\delta}{R} \left( \frac{\delta}{R} + \frac{1}{4\eta_s} \right). \quad (6) \]

Upon replacing the relative viscosity of the suspension, \( \eta_s = \mu_s/\mu_b \), with \( \eta_s = (1 - (\phi/\phi_m))^{-2.0} \) [Krieger and Dougherty (1959)] and the apparent slip layer thickness, \( \delta \), with \( D_p/\delta ) = 1 - (\phi/\phi_m) \) [Kalyon (2005)], \( Q_s/Q \) becomes

\[ \frac{Q_s}{Q} = \frac{1}{1 + \frac{R}{4D_p} \left( 1 - \frac{\phi}{\phi_m} \right)}. \quad (7) \]

Therefore, the application of the apparent slip mechanism to a Newtonian suspension suggests that as \( \phi \rightarrow \phi_m \), \( Q_s/Q \rightarrow 1 \), i.e., the flow approaches plug flow with increasing concentration of the rigid particles and that apparent slip has a negligible effect for relatively small concentration of rigid particles, i.e., \( \phi \rightarrow 0 \), \( Q_s/Q \rightarrow 0 \) since \( R \gg D_p \). Such plug flow behavior is indeed observed during the Poiseuille flow of Newtonian suspensions with increasing \( \phi \), irrespective of the flow rate [Karnis and Mason (1967)].

Thus, with the contributions of the slip of the binder and the apparent slip mechanism the apparent wall slip velocity, \( u_s \), of the suspension becomes:

\[ u_s = \beta_0 \left( \frac{P_m}{P} \right)^{\kappa(\phi)} \tau_w (0.5 + 0.5 \tanh(\alpha(\tau_w - \tau_c))) + \frac{D_p}{m_b} \left( 1 - \frac{\phi}{\phi_m} \right) \tau_w. \quad (8) \]

The rheological characterization of the polymeric binder provides \( \beta_0, s_1, s_2, m_b \), and \( \tau_c \). Upon the characterization of the shear viscosity and the wall slip behavior of the binder of the suspension and the physical properties of the rigid phase (the harmonic mean particle diameter, \( D_p \), volume fraction, \( \phi \), and maximum packing fraction of the particles, \( \phi_m \)) the only unknown in Eq. (8) that describes the wall slip behavior of the suspensions with the same binder is, \( \kappa(\phi) \).

The typical slip velocity, \( u_s \), versus the wall shear stress and pressure behavior generated by Eq. (8) for a model suspension with \( \phi = 0.2 \) is shown in Fig. 2. Various properties of this model suspension are listed in Table 1. The wall slip velocity is affected by both the pressure and the wall shear stress and undergoes significant changes at various critical wall shear stress and pressure combinations representing the weak to strong slip transition that Eq. (8) represents. Figure 2 suggests that if the critical pressure and the wall shear stress conditions (at which the slip behavior is converted from weak to strong slip) were to be encountered at any location along the length of the flow channel, significant changes in the flow boundary condition at the wall will develop.
III. TIME-DEPENDENT MODEL FOR TUBE FLOW OF COMPRESSIBLE SUSPENSIONS SUBJECT TO PRESSURE-DEPENDENT WALL SLIP

Consistent with the earlier literature, the compressible fluid is assumed to be purely viscous, i.e., generalized Newtonian fluid \cite{Hatzikiriakos and Dealy 1992b, Den Doelder et al. 1998, Georgiou 2003}. The inertial effects are assumed to be negligible and isothermal flow in the straight land section of the tube die is considered. The continuity and momentum equations provide \cite{Tang and Kalyon 2008}:

\[
\frac{\partial p}{\partial t} + \frac{\partial}{\partial z}\left[p - p_a + \frac{\rho_a}{\gamma} V\right] = 0,
\]

FIG. 2. Wall slip velocity versus pressure and wall shear stress behavior of model suspension with the parameters given in Table I (\(\phi=0.2\)).

<table>
<thead>
<tr>
<th>Volume fraction of particles, (\phi) (%)</th>
<th>(m) (g/cm³)</th>
<th>(\tau_s) (Pa)</th>
<th>(\kappa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40 000</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0.2</td>
<td>49 279</td>
<td>1142</td>
<td>1.57</td>
</tr>
<tr>
<td>0.4</td>
<td>67 640</td>
<td>4571</td>
<td>2.14</td>
</tr>
<tr>
<td>0.6</td>
<td>133 666</td>
<td>10 286</td>
<td>2.71</td>
</tr>
</tbody>
</table>

Note: \(\tau_s = \alpha_1 \left[\frac{\phi}{\Phi}\right]^2\) with \(\alpha_1 = 14 000\) Pa, \(m(\phi) = \frac{1 - \phi}{\phi_m}\), with \(k\phi_m = 0.62\), \(\beta_m = \frac{D_p}{m_p} \left[1 - \frac{\phi}{\phi_m}\right]\) with \(D_p = 10\) μm, and \(\kappa = \alpha_2 \left[\frac{\phi}{\phi_m}\right] + 1\) with \(\alpha_2 = 2\), \(s(\phi) = s_b(1 - \phi)\) and \(s_b = 10^{-5}\) s²/m².
\[ \frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial \chi V^2}{\partial z} + \frac{2\tau_w}{(s(p-p_a)+p_a)R} + \frac{1}{(s(p-p_a)+p_a)} \frac{\partial p}{\partial z} = 0, \]  

where \( t \) is the time, \( V \) is the cross-section-averaged \( z \) velocity, the density is given by \( p=s(p-p_a)+p_a \), i.e., \( p \) and \( p_a \) are the density values of the fluid at pressures \( p \) and \( p_a \), respectively, \( s \) is the compressibility coefficient, and \( \chi=(1/(AV^2))\int f u^2 dA \). (1/2)(\( \partial V^2 / \partial z \)) was assumed to be negligible for creeping flow conditions. The compressibility coefficient of the suspension, \( \varsigma \), is a function of the concentration of the rigid particles, i.e., \( \varsigma=\varsigma_b(1-\phi) \).

The boundary conditions at the entry and exit planes of the tube are

\[ V = V_0, \quad z = 0. \]
\[ p = p_a, \quad z = L. \]

Under creeping flow conditions in which the \((\partial V/\partial t)\) term can be considered to be negligible in comparison to the other two terms in Eq. (10), the wall stress, \( \tau_w \), can be determined from the prevailing steady local pressure gradient, \((dp/\partial z)\), i.e., \( \tau_w = -(R/2)((dp/\partial z)) \). The relationship between the mean velocity, \( V \), versus the pressure gradient, \((dp/\partial z)\), for a Hershel–Bulkley-type viscoplastic suspension subject to a wall slip velocity, \( u_s \), under steady state flow conditions is [Kalyon et al. (1993)]:

\[ V = \begin{cases} 
\frac{16\tau_y}{R^2m^{1/n}(1+1/n)(2+1/n)} - \frac{16}{m^{1/n}(1+1/n)(3+1/n)} - \tau_y - \frac{R}{2} \left( \frac{dp}{dz} \right)^{2+1/n} & \left( \frac{dp}{dz} \right) < \tau_y, \\
\frac{2}{m^{1/n}(1+1/n)} - \tau_y - \frac{R}{2} \left( \frac{dp}{dz} \right)^{3+1/n} & \left( \frac{dp}{dz} \right) \geq \tau_y, \\
\tau_y + u_s, & \left( \frac{dp}{dz} \right) \leq \tau_y,
\end{cases} \]

where \( \tau_y, m \) and \( n \) are the yield stress, consistency index and the power law index of the Herschel–Bulkley fluid. A finite difference method was used for the numerical solution of the time and location-dependent pressure, \( p \), and mean velocity, \( V \), distributions in tubular flow, and the details of the numerical solution and the stability criteria used can be found in Tang and Kalyon (2008).

Typical numerical results of the solutions of the time-dependent tube flow, i.e., the solutions of Eqs. (9)–(13), were obtained first for a model polymer melt and its three density-matched suspensions “model suspensions.” The properties of the model melt and its suspensions are provided in Table I. For the model suspensions, it is assumed that the ratio of the consistency index of the suspension, \( m(\phi) \), over the consistency index of the binder, \( m_p \), follows a modified Krieger and Dougherty model \((m(\phi)/m_p)=(1-(\phi/\phi_m)^{-k\phi_m})^{-1} \) converted from that of the behavior of suspensions with Newtonian binders [Krieger and Dougherty (1959)] and assuming that the power law indices for the model melt and its suspensions are similar (Table I). The yield stress of a suspension can be taken to vary with the square of the concentration of particles [Friend and Hunter (1971); Mewis (1976)]. Here it is assumed that the yield stress depends on the square of \((\phi/\phi_m)\).
To be able to make inferences on how the pressure coefficient of slip, κ, depends on the concentration of particles, one needs to elucidate the mechanism of the experimentally observed pressure dependence of wall slip. Two major effects associated with the time necessary to establish a fully developed apparent slip condition in the die and air entrainment into the fluid can lead to the experimentally observed decrease of the wall slip velocity values with increasing pressure. Aral and Kalyon (1994) have investigated the time and temperature dependent development of wall slip of concentrated suspensions in steady torsional flow. This experimental study has shown that there is a characteristic time necessary to reach a steady wall slip velocity and that the characteristic time necessary to reach the steady wall slip velocity of the suspension decreases with increasing shear rate.

One possible mechanism to the development of such characteristic times can be linked to the particle migration effects associated with shear rate gradients [Gadala-Maria and Acrivos (1980); Leighton and Acrivos (1987)]. In channel flow the distance necessary to reach fully developed concentration distributions would scale with the particle diameter/channel gap ratios [Phillips et al. (1992); Nott and Brady (1994)]. The formation of the apparent slip layer would then be coupled to such particle migration effects [Jana et al. (1995); Allende and Kalyon (2000)]. These observations suggest that the entry region of a die, at which the pressure is highest, is also most likely to be devoid of wall slip since the slip condition requires a finite time/distance to be established. However, no data are available on the effects of the particle concentration on the kinetics of wall slip development to permit an inference on κ(ϕ) to be made.

The air entrained into the suspension also significantly affects the wall slip behavior. Air is entrained during processing of suspensions especially during the compounding of particles with the polymeric binder [Kalyon et al. (1991)] and significantly affects the shear viscosity, wall slip, and processability of the suspension [Aral and Kalyon (1995)]. The investigation of the flow of a concentrated suspension through a transparent rectangular slit die has revealed that bubbles of air reach the surface of the die, flatten and then release in a cyclic fashion to generate an additional lubrication mechanism at the wall [Kalyon et al. (1995)]. Imaging experiments with various other fluids have also revealed the formation of a vapor layer (some as nanobubbles) at the wall [Tyrrell and Attard (2001)]. Since the nucleation and the growth of the air bubbles would occur as a function of bulk pressure of the fluid, the highest wall slip velocities are expected to occur at the exit of the die where the pressure is the lowest, thus providing an important mechanism for the wall slip velocity to increase with decreasing pressure. The amount of air entrained increases significantly upon the compounding of rigid particles into a melt [Lu et al. (2004)] and should therefore increase with increasing concentration of particles. This suggests that the effect of pressure on wall slip of concentrated suspensions would be more pronounced with increasing concentration of particles. Thus, it is reasonable to assume that κ would increase with increasing ϕ/ϕₘ. For the model suspension of Table I it is assumed that the exponent κ increases linearly with ϕ/ϕₘ. The κ(ϕ) values can be determined from squeeze flow, used in conjunction with the inverse problem solution methodologies.

Typical results are shown in Figs. 2–4 for the model suspension at ϕ=0.2. Figure 3 shows the mean velocity, V, slip velocity, uₛ, the wall slip velocity associated with the apparent slip mechanism, uₛₛ, pressure, p, and wall shear stress, τₛₛ, distributions for the flow occurring at a mass flow rate of 3.85×10⁻⁴ kg/s through a circular tube with a diameter of 1 mm and a length, L, of 50 mm. The mean velocity, V, slip velocity, uₛ, the contribution of the apparent slip mechanism, uₛₛ, pressure, p, and wall shear stress, τₛₛ, all become time independent at all locations along the length of the tube [Fig. 3(b) for z
Under steady state, the pressure gradient \( dp/dz \) and the wall shear stress, \( \tau_w \), decrease monotonically with distance along the length of the tube, reaching minima at the die exit [Fig. 3(b)]. The contribution of the wall slip to the overall flow rate is relatively small adjacent to the entry region of the die.

Figure 3(b) also indicates that the wall slip velocity of the model suspension with \( \phi = 0.2 \) increases monotonically with increasing distance at the mass flow rate of \( 3.85 \times 10^{-4} \) kg/s, with the maximum slip velocity observed at the exit. In fact, at the exit of the tube the slip velocity becomes equal to the mean velocity of the suspension, i.e., plug flow occurs at the exit plane. The plug flow formation at the exit plane removes the stress singularity that is presumed to exist at the exit of the tube. Such singularities would arise upon the conversion of the generally parabolic velocity distribution in the tube to plug flow of the extrudate immediately upon exit from the tube.

The contribution of the apparent slip mechanism to overall slip behavior of the suspension, \( u_{sa} \), decreases with increasing distance, as the wall slip of the binder becomes the major source of the wall slip of the suspension with increasing distance, i.e., decreasing pressure in the die [Fig. 3(b)]. Both the compressibility and pressure dependence of the slip coefficient of the suspension increase the mean velocity with increasing distance towards the exit of the cylindrical die, however, the effect of compressibility is not significant in comparison to the effect of pressure on the slip coefficient for the conditions investigated here. The wall shear stress values are greater than the critical shear stress at \( L/2 \). 

FIG. 3. Time and location-dependent distributions of pressure, \( p \), wall shear stress, \( \tau_w \), mean velocity, \( V \), slip velocity, \( u_s \), and the contribution of the apparent slip mechanism to wall slip, \( u_{sa} \), for a mass flow rate of \( 3.85 \times 10^{-4} \) kg/s for \( \phi = 0.2 \) (Table I). (a) Time dependence at \( L/2 \). (b) Location dependence under steady state.
which weak to strong slip transition occurs (80 kPa for the model suspension of Table I) along the entire length of the tube wall under the steady flow conditions achieved (Fig. 3).

On the other hand, when the mass flow rate of the same suspension ($\phi=0.2$) is decreased sufficiently, for example, to $2.76 \times 10^{-6}$ kg/s, unsteady flow behavior is observed. The slip velocity and the mean velocity change significantly with location and time and a steady state cannot be reached, regardless of the conditions employed in numerical analysis (Fig. 4). At this lower flow rate, the wall shear stress values cross the critical shear stress threshold. The imposition of the Rankine-Hugoniot condition indicates that the solution of the problem stated by Eqs. (9) and (10) cannot have two steady states separated by a discontinuity “jump” in slip velocity, i.e., a transition between weak and strong slip at the wall [Tang and Kalyon (2008)]. This transition appears to have taken place under the flow conditions used to generate the unsteady results shown in Fig. 4. The time-averaged wall shear stress and the pressure values behave similar to the steady case and decrease monotonically with axial distance.

The predicted flow curves of the model binder and its suspensions ($0 \leq \phi \leq 0.6$) are shown in Fig. 5. The predicted steady and unsteady flow conditions are delineated over a broad range of apparent shear rates. The reported values of the wall shear stress and the flow rate (apparent shear rate) for the unsteady cases are time-averaged values. For the pure polymeric binder it is seen that most of the flow conditions that make up the flow curve, i.e., the apparent shear rate range of $20–4000$ s$^{-1}$, give rise to unsteady cases. With increasing concentration of rigid particles the range of apparent shear rates over
which the flow becomes unsteady decreases. It is clear that the results reported in Figs. 2–5 would be affected by the assumed parameters for the model melt and suspensions. However, provided that the melt exhibits wall slip in conjunction with a critical shear stress for the onset of strong wall slip and such wall slip is pressure dependent, the incorporation of the particles into the polymer melt would have a stabilizing effect on flow, as Fig. 5 suggests.

It is interesting to note that there are some polymers and elastomers that exhibit a no slip condition at the wall for a broad range of apparent shear rates and wall shear stresses, presumably on the basis of the wall shear stress values not reaching their respective critical wall shear stress values (at which the weak to strong slip transition of the melt takes place) [Kalyon and Gevgilili (2003)]. It may be hypothesized that the contiguous no slip condition at the wall under such flow conditions would lead to steady flow, as indeed observed [Kalyon and Gevgilili (2003)]. However, if particles were added to such polymer melts with relatively high critical shear stress values, significant increases in the wall shear stress values would occur under similar apparent shear rate conditions to possibly enable the reaching of the critical shear stress value of the melt at some location along the length of the die. Under the right conditions this could lead to the disruption of the contiguous stick condition along the length of the wall and could give rise to unsteady behavior under similar apparent shear rate conditions. Such unstable flow behavior was indeed observed experimentally for the highly filled suspensions of an oxetane binder, which itself exhibits stick at the wall and no flow instabilities over a broad range of apparent shear rates [Tang and Kalyon (2004a)]. However, detailed characterization results are not available for this oxetane binder and its suspensions and numerical analysis of the flow of this interesting binder and its suspensions could not be carried out.

To validate the mechanisms elucidated above, a direct comparison between experimental and numerical simulation results is desirable. To enable a comparison, the shear viscosity and wall slip behavior of the PDMS and its suspensions with rigid spherical glass particles of the study of Birinci and Kalyon (2006) were characterized as outlined next. This is followed by a comparison of the available experimental flow curves and flow instability behavior of the PDMS/glass suspensions with the simulation results.

FIG. 5. Flow curves and predicted steady and unsteady flow regimes of the model binder and its suspensions of Table I (filled circles—unsteady, open circles—steady).

TIME-DEPENDENT TUBE FLOW OF SUSPENSIONS

1079
IV. PARAMETERS FOR SHEAR VISCOSITY AND PRESSURE-DEPENDENT WALL SLIP OF SUSPENSIONS OF PDMS WITH GLASS SPHERES

A. Materials

GE Silicones (SE-30) manufactures the binder used in the experimental study, i.e., the poly(dimethyl siloxane), PDMS. It has a density of 958 kg/m³. The compressibility coefficient, $\zeta$, of the PDMS, was determined to be $1.4 \times 10^{-6}$ (s²/m²), which agreed with the available pressure-volume-temperature data of PDMS fitted with the Sanchez and Lacombe Lattice Fluid Model [Brandrup et al. (1999)]. The rigid particles incorporated into the PDMS are spherical and hollow glass particles with a specific gravity of 1.098 and an arithmetic mean particle diameter of 12 μm (Potters Industry, Inc). The suspension density at ambient pressure increases linearly from 987 at $\phi=0.1$ to 1073 kg/m³ at $\phi=0.4$. The maximum packing fraction of the glass particles was estimated as 0.69 [Yaras (1995)]. The particles were compounded into the PDMS using a Haake intensive mixer. The attrition of the hollow glass particles upon mixing and rheological characterization was determined to be negligible.

B. Experimental apparati and procedures

An Advanced Rheometric Expansion System rheometer, originally from Rheometric Scientific, Inc., Piscataway, NJ (currently TA Instruments), was utilized in conjunction with steady torsional flow and small-amplitude oscillatory shear flows using cone-and-plate and parallel-disk configurations. The environmental chamber was equipped with an imaging window and auxiliary optics for the continuous monitoring of the free surface of the specimen [Aral and Kalyon (1994); Gevgilili and Kalyon (2001); Gevgilili (2003)]. A high-speed camera, capable of recording at filming speeds, which can be as high as 2000 frames per second, was part of the setup to allow the monitoring of the free surface of the specimen which was trimmed flat to avoid surface tension effects. During steady torsional flow a straight-line marker was placed on the edges of the cone/plate and the free surface of the polymer melt to enable the determination of the conditions under which wall slip occurs at the edge of the specimen [Kalyon et al. (1993); Aral and Kalyon (1994); Gevgilili and Kalyon (2001)]. The discontinuities that develop between the surfaces of the disks of the rheometer and the bulk of the melt suggest the initiation of strong wall slip. From this experiment the critical temperature-independent shear stress for the onset of strong wall slip of the PDMS binder was determined to be 70 kPa [Kalyon and Gevgilili (2003)].

The dependence of the wall slip coefficient on pressure precludes the use of pressure-driven rheometers (capillary and rectangular slit) and associated methodologies for the characterization of the shear viscosity and wall slip behavior of the PDMS and its suspensions. This was also recognized by Dealy and co-workers and the magnitude of complex viscosity values were utilized to represent slip-free shear viscosity values on the basis of the Cox–Merz rule. In our study the parameters of the shear viscosity material function of PDMS were obtained by using the K-BKZ integral-type constitutive equation [Bernstein et al. (1963); Wagner (1976); Osaki (1976); Laun (1978); Kalyon et al. (1988)] in conjunction with small-amplitude oscillatory shear and steady torsional flow data, which could be collected in the absence of wall slip [Tang and Kalyon (2008)]. It was convenient to fit the shear viscosity versus the shear rate behavior of the PDMS with two sets of power-law parameters, i.e., $m=19 000$ Pa s$^{0.37}$, $n=0.37$ for shear rates less than 40 s⁻¹ and $m=31 000$ Pa s$^{0.21}$, $n=0.21$ for shear rates greater than 40 s⁻¹. The torque data from the steady torsional flow of PDMS collected at various nominal shear rates were used for the determination of the slip parameters $\beta_0$ and $s_1$ as 5
The squeeze flow data of the PDMS were used for the determination of the exponent $\kappa = 0.7$ [Tang and Kalyon (2008)].

The experimentally collected steady torsional data, i.e., the steady torque versus apparent shear rate values of suspensions of PDMS with spherical particles ($\phi = 0.1, 0.2,$ and $0.4$) (Fig. 6) and the squeeze flow data (Fig. 7) were then used for the determination of the yield stress, $\tau_y(\phi)$, the consistency index, $m(\phi)$ and $\kappa(\phi)$ exponent that defines the pressure dependence of the wall slip coefficient of the PDMS suspensions (Table II) upon the solution of the associated inverse problem [Tang and Kalyon (2008)]. It was assumed that the power law index, $n$, is independent of the concentration of the particles, $\phi$ and that $\tau_y(\phi) = \alpha_1(\phi/\phi_m)^2$ and $\kappa(\phi) = \alpha_2(\phi/\phi_m) + 0.7$, where 0.7 is the $\kappa$ value for the pure binder, i.e., PDMS.

Re squeeze flow the finite element method, based source codes for the analysis of the squeeze flow of viscoplastic fluids subject to wall slip [Lawal et al. (2000); Tang and Kalyon (2004b); Kalyon and Tang (2007)] were modified to allow the imposition of pressure dependent slip coefficients $\kappa(\phi)$ according to Eq. (8).

$\times 10^{-16} \text{ m}^2/(\text{s Pa}^{3.1})$ and 3.1, respectively. The squeeze flow data of the PDMS were used for the determination of the exponent $\kappa = 0.7$ [Tang and Kalyon (2008)].

![FIG. 6. Torque, $T$, vs. the apparent shear rate in steady torsional flow. Diamonds: pure PDMS, squares: $\phi = 0.1$, triangles: $\phi = 0.2$, circles: $\phi = 0.4$. Open symbols: predictions, filled symbols: experimental data.](image)

![FIG. 7. Normal force, $F$, vs. gap, $h$, during squeeze flow of PDMS suspension with spherical hollow glass spheres ($\phi = 0.1$) as a function of the plate velocity.](image)
The typical reasonable agreement between the steady torque versus apparent shear rate data collected with the steady torsional flow and those predicted on the basis of the parameters determined with the inverse problem solution methodologies is shown in Fig. 6. The comparisons of the experimentally determined normal force, $F$, during squeeze flow of the PDMS/glass suspension $\phi=0.1$ as a function of the gap, $h$, at various crosshead speeds and the predictions obtained from FEM on the basis of the parameters determined using the inverse problem solution methodologies are shown in Fig. 7. The agreement is again reasonable.

The typical reasonable agreements shown in Figs. 6 and 7 between the experimental steady torsional and squeeze flow data and the predictions on the basis of the best fit inverse problem solution of $\gamma$, $\phi$, and $m$ are reasonable and suggests that the simple relationships used to describe the shear viscosity and the apparent wall slip of the suspensions are acceptable. The parameters determined for the PDMS and its suspensions are tabulated in Table II. These parameters were used in the solution of the time-dependent circular tube flow, as reported in the next section.

An Instron capillary rheometer was employed to study the development of extrudate distortions of the PDMS suspensions upon exit from the capillary dies [Birinci and Kalyon (2006)]. The shapes of the extruded samples, immediately upon exit from the die, were captured using a high-speed camera. The temperature distributions of the extrudates emerging from the die were also monitored continuously using a ThermaCam thermal imaging camera. These data, collected for the PDMS suspensions ($\phi=0.1$, 0.2, and 0.4), were compared with the predictions of the numerical model for the flow curves as well as the limits of steady and unsteady behavior. It should be noted that the capillary flow data collected for the characterization of the flow curves, as well as for the analysis of the flow instability behavior of the suspensions of PDMS, were not used in the determination of the parameters of wall slip and the shear viscosity of the PDMS and its suspensions with glass spheres.

### Table II

Parameters for shear viscosity and wall slip of PDMS and its suspensions with glass spheres for which the simulation results are reported in Figs. 8–14.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\tau_c$, Pa</th>
<th>$\gamma$</th>
<th>$\beta_0$, m s$^{-1}$, Pa s$^{-1}$</th>
<th>$s_1$</th>
<th>$\tau_c$, Pa</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.7</td>
<td>$5 \times 10^{-16}$</td>
<td>3.1</td>
<td>70 000</td>
<td>20</td>
</tr>
<tr>
<td>0.1</td>
<td>1 250</td>
<td>1.03</td>
<td>\nodata</td>
<td>\nodata</td>
<td>\nodata</td>
<td>\nodata</td>
</tr>
<tr>
<td>0.2</td>
<td>5 000</td>
<td>1.36</td>
<td>\nodata</td>
<td>\nodata</td>
<td>\nodata</td>
<td>\nodata</td>
</tr>
<tr>
<td>0.4</td>
<td>20 000</td>
<td>2.0</td>
<td>\nodata</td>
<td>\nodata</td>
<td>\nodata</td>
<td>\nodata</td>
</tr>
</tbody>
</table>

Shear rate $<40$ s$^{-1}$

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>Consistency index, $m_a$, Pa s$^{0.5}$ or $m$, Pa s$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>19 000</td>
</tr>
<tr>
<td>0.1</td>
<td>20 770</td>
</tr>
<tr>
<td>0.2</td>
<td>23 180</td>
</tr>
<tr>
<td>0.4</td>
<td>32 220</td>
</tr>
</tbody>
</table>

Shear rate $>40$ s$^{-1}$

$\tau_c=\alpha_1\left(\frac{\phi}{\phi_m}\right)^2$ with $\alpha_1=59 535$ Pa, $\beta_0=\frac{D_p}{m^a}\left[1-\frac{\phi}{\phi_m}\right]$ with $D_p=12$ $\mu$m, and $\gamma=\alpha_2\left(\frac{\phi}{\phi_m}\right)+0.7$ with $\alpha_2=2.3$
V. RESULTS AND DISCUSSION

The governing Eqs. (9) and (10) were solved to simulate the time-dependent tube flow of PDMS suspensions incorporated with 10%, 20%, and 40% by volume glass spheres. Three capillary dies with diameters of 0.83 mm, 1.5 mm, and 2.5 mm and a constant length over the diameter ratio, $L/D$, of 40 were used.

Figure 8(a) shows the time-dependent distributions of pressure, $p$, mean velocity, $V$, slip velocity, $u_s$, and wall shear stress, $\tau_w$, (for $\phi=0.1$) at a location next to the exit ($z=0.95$ L) under the conditions of $D=2.5$ mm, $L/D=40$, $V_0=1$ mm/s (apparent shear rate of 3.2 s$^{-1}$), together with $I=101$, $\text{CFL}=0.25$, and $\text{Von}=0.25$ ($I$—number of mesh nodes, CFL—Courant–Friedrichs–Levy number, Von—von Neumann number—[Tang and Kalyon (2008)]. A steady state is predicted to be reached at this apparent shear rate [Fig. 8(a)]. Figure 8(b) shows the steady state distributions of pressure, $p$, mean velocity, $V$, slip velocity, $u_s$, and wall shear stress, $\tau_w$, over the length of the tube. Under the steady state flow conditions the wall shear stress increases slightly in the axial, $z$ direction [Fig. 8(b)]. The wall shear stress values along the entire length of the tube are smaller than 70 kPa at which the pressure-dependent transition between the weak to strong slip of the binder takes place. Consequently, the wall slip velocity values, associated with the apparent slip mechanism, do not change significantly over the length of the tube [Fig. 8(b)]. The contribution of the slip of the binder to the slip velocity, $u_s$, of the suspension is negligible under this flow condition and geometry.

The numerical solution was determined to be independent of the grid spacing and the time interval employed by systematically altering the number of mesh nodes, $I$, and the
values of the Courant–Friedrichs–Levy, CFL and von Neumann, Von, numbers. The value of \( \alpha \) was kept in between 1 and 20 and over this range the slip behavior was not sensitive to the value selected. Overall, the simulation results reveal no fluctuations in pressure, wall shear stress, mean velocity, and slip velocity at such relatively low flow rates for \( \phi=0.1 \).

For the same PDMS suspension \( \phi=0.1 \) steady flow is also predicted at the relatively high apparent shear rate range, i.e., \( \geq 1200 \text{ s}^{-1} \) with typical results shown in Fig. 9 at the apparent shear rate of \( 2700 \text{ s}^{-1} \) (for a tube die with \( D=0.83 \text{ mm}, L/D=40, V_0 =0.28 \text{ m/s}, \) and \( I=101 \)). Under such high apparent shear rate conditions the shear stress values at the wall are all greater than the critical wall shear stress of the PDMS binder at which the transition from the stick to the slip condition (weak to strong slip of the binder) occurs. This gives rise to wall slip velocity values that are relatively high along the entire length of the tubular die. The wall slip velocity values of the suspension increase monotonically in the \( z \) direction as affected by the increasing slip coefficient of the binder, \( \beta_b \), with decreasing pressure [Eq. (2)] to a maximum at the exit, where the flow becomes a plug flow, that is, \( V=u_s \).

The steady behavior presented in Figs. 8 and 9 is typical for apparent shear rates that are either \( <20 \text{ s}^{-1} \) or \( >1200 \text{ s}^{-1} \) for \( \phi=0.1 \). However, over the intermediate apparent shear rate range of \( 20–1200 \text{ s}^{-1} \), steady solutions could not be reached for \( \phi=0.1 \) (see Fig. 10 for an apparent shear rate of \( 160 \text{ s}^{-1} \) for a tubular die with \( D=1.5 \text{ mm}, L/D =40, V_0=0.03 \text{ m/s}, \) and using \( I=101 \)). Over the intermediate apparent shear rate range of

**FIG. 9.** Time and location dependent distributions of pressure, \( p \), wall shear stress, \( \tau_w \), mean velocity, \( V \), and wall slip velocity, \( u_s \), at the apparent shear rate of \( 2700 \text{ s}^{-1} \) for \( \phi=0.1 \). (a) Time dependence at \( z=0.95 \text{ L} \). (b) Location dependence under steady state.
20–1200 s\(^{-1}\), the mean velocity and the wall slip velocity values exhibit time-dependent oscillations that do not dampen to a steady state, regardless of how long the simulations are carried out over broad ranges of the simulation parameters, \(I\), CFL and \(V\).

The wall shear stress distributions for conditions that did not give rise to a steady solution indicated that for these cases the critical wall shear stress and pressure combinations (at which weak to strong wall slip transitions occur) were crossed (Fig. 10). Thus, overall flow conditions under which the pressure and wall shear stress values consistently stay above or below the critical conditions generate steady solutions. On the other hand, the conditions that give rise to the crossing of the critical conditions, anywhere along the length of the die, do not allow steady solutions.

The summary of the computations for \(\phi=0, 0.1, 0.2\), and 0.4 for the tube diameters of 0.83 mm, 1.5 mm, and 2.5 mm at the constant tube length/diameter, \(L/D\), ratio of 40 are plotted in Figs. 12–14. For comparison, experimental data of Birinci and Kalyon (2006) are also included in Figs. 12–14, along with the typical extrudate shapes recorded immediately upon the exit of the extrudate from the die. The experimental and theoretical flow curves of the suspensions of PDMS agree with each other and the numerical data capture well the trends in the development of flow instabilities for the suspensions.

For \(\phi=0.1\) and 0.2 the simulations suggest stable flow at the low and high apparent shear rates and unstable flow in the intermittent apparent shear rate ranges (Fig. 11). Furthermore, with increasing concentration of particles the introduction of the apparent slip mechanism increases the range of apparent shear rates, over which steady flow is observed (Fig. 11). The computations for the PDMS suspensions suggest that conditions

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**FIG. 10.** Time and location dependent distributions of pressure, \(p\), wall shear stress, \(\tau_w\), mean velocity, \(V\), and wall slip velocity, \(u_s\), at an apparent shear rate of 160 s\(^{-1}\) of PDMS suspension with glass spheres (\(\phi=0.1\)). (a) Time dependence at \(L/2\). (b) Typical location dependence under unsteady state conditions.
which give rise to a stable slip condition at the wall are predicated on whether the wall shear stress and pressure along the length of the die are all consistently above or below the critical conditions at which transition between weak to strong slip occur. The range of apparent shear rates over which unstable flow is predicted decreases with increasing volume loading level of rigid particles, $\phi$.

The experimental data associated with extrudate distortions presented in Figs. 12–14 generally agree with the stability predictions. In these figures the region in between the two dotted lines represent the range of apparent shear rates over which extrudate distortions are experimentally observed.
and bulk distortions of the extrudates are clearly observed in the apparent shear rate range of 10–500 s⁻¹. At apparent shear rates that are below or above this range the extrudates emerging from the three capillary dies are all smooth (Fig. 12). Upon the increase of the concentration of the rigid particles to 20% by volume, the distortions in the shapes of the extrudates emerging from the three capillaries become less pronounced and disappear for the greatest capillary diameter of 2.5 mm. The range of apparent shear rates over which surface distortions of the extrudates can be identified at φ=0.2 moves to the higher apparent shear rate range of 100–1000 s⁻¹ (Fig. 13). Finally at φ=0.4 none of the extrudates exhibits surface or bulk distortions. Except for two points at the lower apparent shear rate range the simulations also predict that stable flow will prevail over the same apparent shear rate range for φ=0.4 (Fig. 14).

FIG. 13. Comparisons of the experimental flow curve of PDMS suspension (φ=0.2) with the numerical simulations: Diamonds: D=0.83 mm, squares: D=1.5 mm, triangles: D=2.5 mm, filled symbols: points that are predicted to be unstable, open symbols: points that are predicted to be stable, open symbols with crosses experimental data; dotted lines represent the lower and upper bounds of apparent shear rates over which extrudate distortions are experimentally observed.

FIG. 14. Comparisons of the experimental flow curve of PDMS suspension (φ=0.4) with the numerical simulations: Diamonds: D=0.83 mm, squares: D=1.5 mm, triangles: D=2.5 mm, filled symbols: points that are predicted to be unstable, open symbols: points that are predicted to be stable, open symbols with crosses experimental data; extrudate distortions were not observed at φ=0.4.
VI. CONCLUDING REMARKS

The roles played by the compressibility and wall slip in the simple shear flow of suspensions of concentrated suspensions are elucidated in conjunction with the apparent slip mechanism of suspensions. The apparent slip mechanism was modified to include the pressure dependent wall slip of the binder phase.

The numerical simulation results of the initial value problem associated with the start-up tube flow indicate that steady solutions are obtained for the melt and its suspensions only under conditions under which a jump associated with the flow boundary condition (typically a transition from weak to strong wall slip) does not take place anywhere along the length of the die. When such transitions occur no steady solution becomes possible. Numerical simulation results obtained for two different series of suspensions with binders which exhibit wall slip indicate that the range of apparent shear rates over which the flow remains stable increases with increasing concentration of particles. This finding agrees with experimental findings. Furthermore, the detailed rheological characterization of a PDMS melt and its three suspensions with spherical glass particles, followed by numerical simulation of their tube flow behavior, generated flow curves that generally agreed with the experimental flow curves.

There are a number of shortcomings in the numerical analysis presented here. In the simulations the converging flow from the reservoir to the straight land section of the capillary dies was not included in the numerical analysis. Furthermore, although the apparent shear rate regions over which steady tube flow occurred could be predicted for the polymer melt and its suspensions, the ignoring of the viscoelasticity of the melt and its suspensions, and the lack of the basic understanding of the role played by a pressure dependent slip condition on viscoelastic flow precluded the detailed analysis of the unsteady region. The expansion of the numerical method to two-dimensional analysis, the inclusion of the inertia term, inclusion of the converging region in the capillary die, and the consideration of the extrudate upon exit from the die are additional areas that need to be followed on.

ACKNOWLEDGMENTS

The authors are grateful to Ms. E. Birinci and Dr. H. Gevgilili of Stevens Institute of Technology for generating the experimental data on the capillary flow of PDMS and its suspensions, and the small-amplitude oscillatory shear and shear stress growth and relaxation data and analyses for the PDMS, respectively. The valuable suggestions and input of Professor M. Denn of Levich/CUNY and Professor J. Dealy of McGill are gratefully acknowledged.

References


